NIS Grade 11 Physics

Introduction to Oscillations and Simple Harmonic Motion

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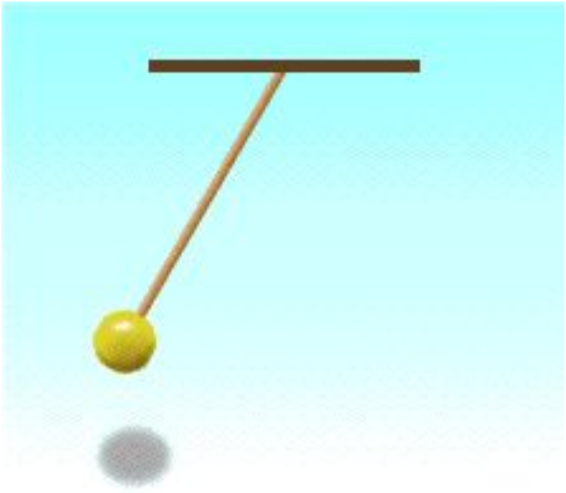


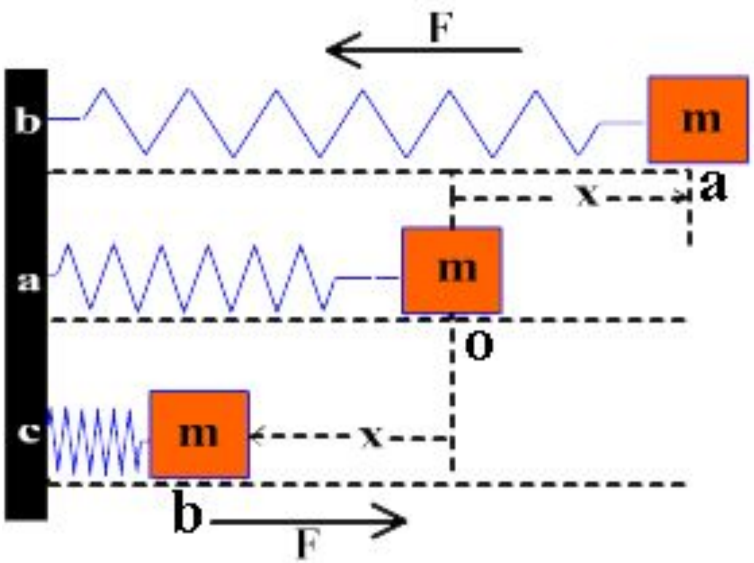
Objectives:

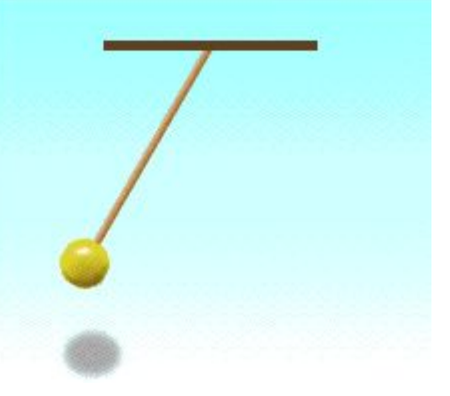
- Describe examples of free oscillations.
- Describe the conditions of Simple Harmonic Motion.
- Apply Hooke’s Law for objects moving with simple harmonic motion.
- Describe the motion of pendulums and calculate the length required to produce a given frequency.

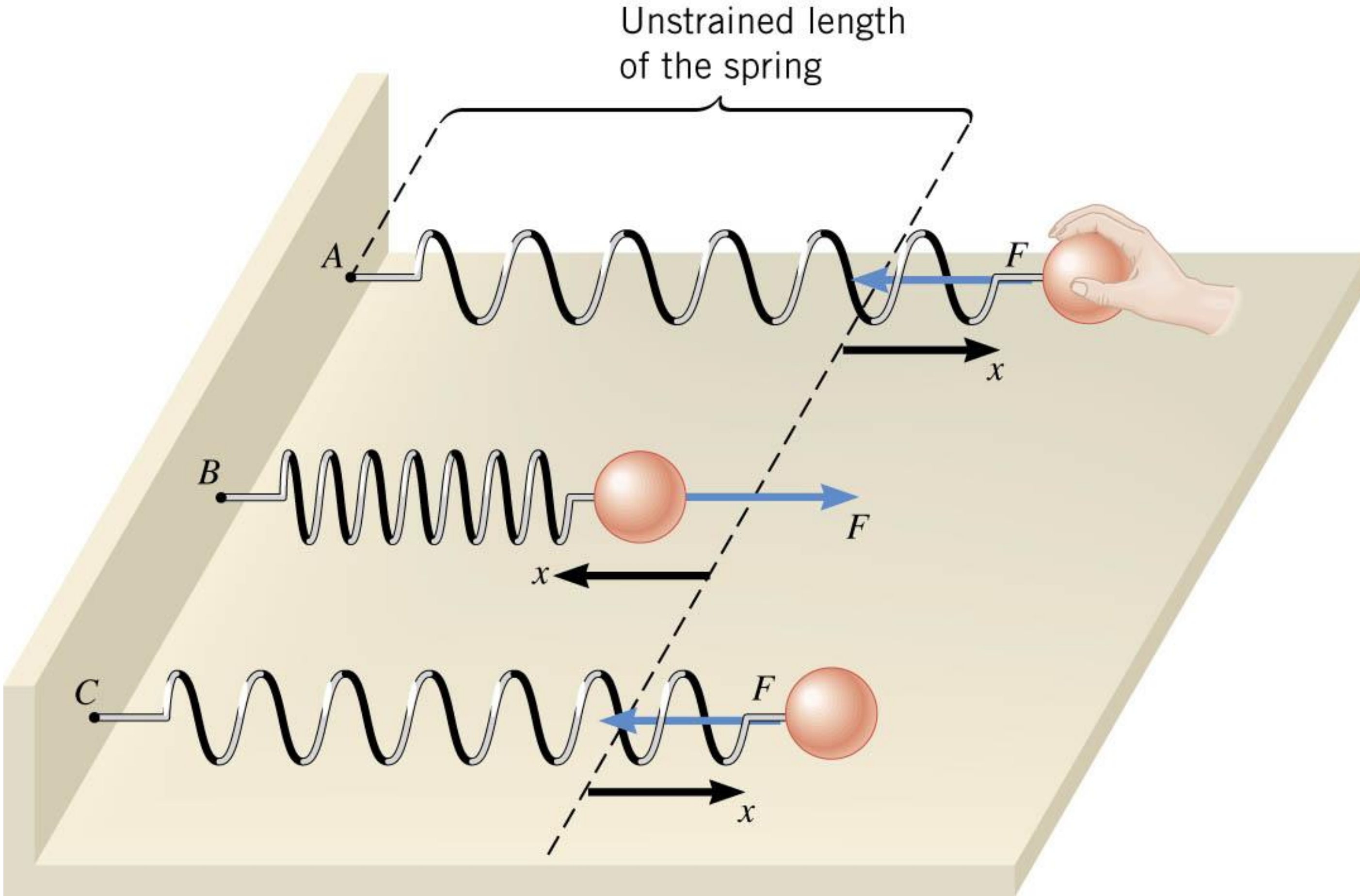
Definition: Simple Harmonic Motion

Any periodic back and forth motion produced by a restoring force that is **directly proportional** to the displacement **and in the opposite direction**. The displacement centers around an equilibrium position.

$$F_s \alpha \; x$$



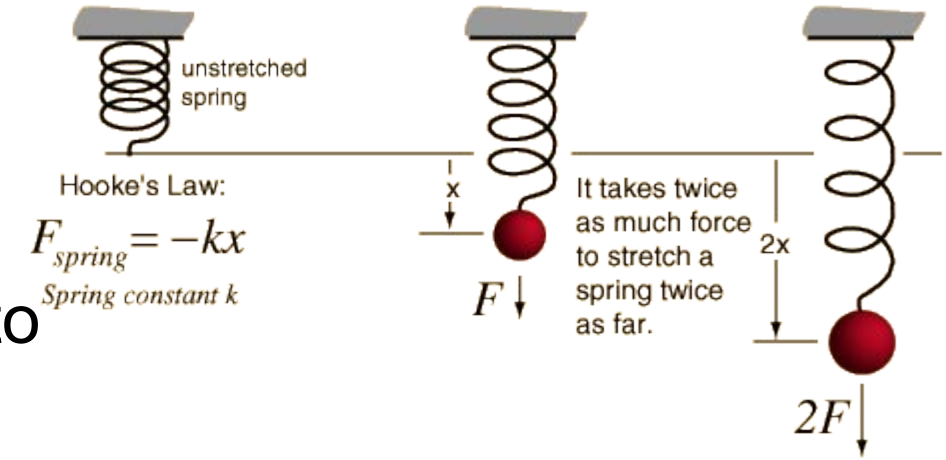




When the restoring force has the mathematical form $F = -kx$, the type of friction-free motion illustrated in the figure is designated as "simple harmonic motion."

Springs – Hooke's Law

One of the simplest type of simple harmonic motion is called **Hooke's Law.** This is primarily in reference to SPRINGS.



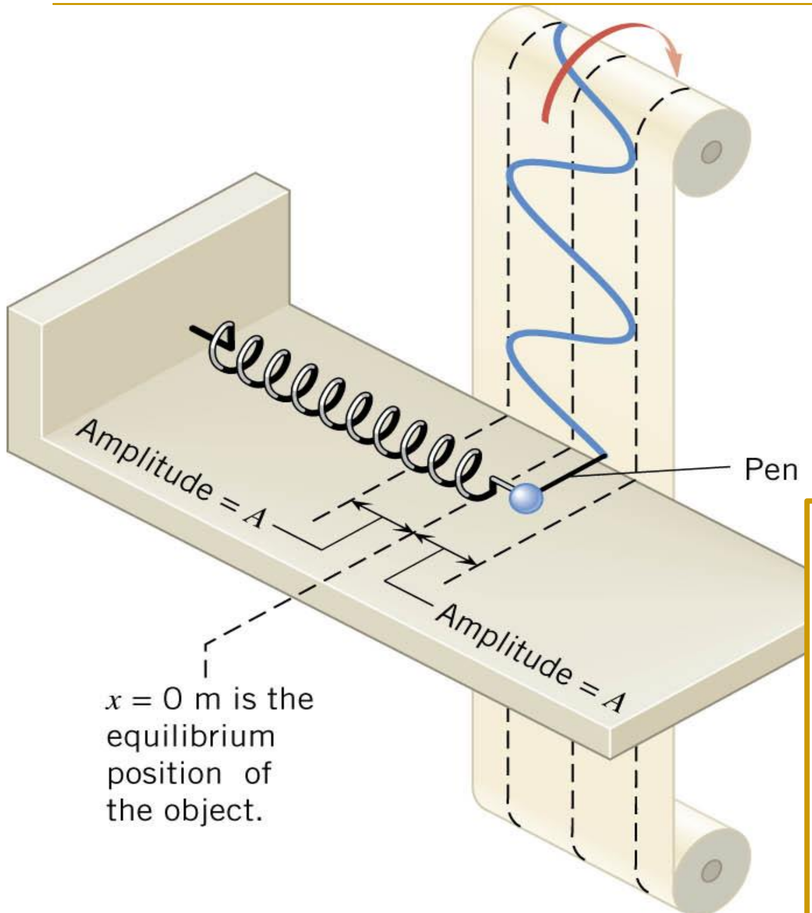
$$F_s \, \alpha \, x$$

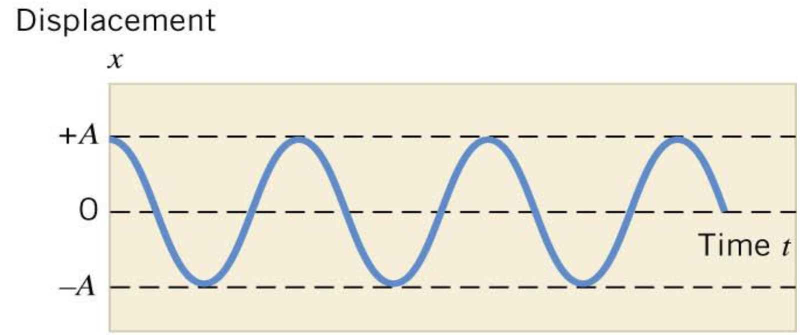
$$k = \text{Constant of Proportionality}$$

$$k = \text{Spring Constant(Unit : N/m)}$$

$$F_s = kx \quad or \quad -kx$$

The negative sign only tells us that "F" is what is called a RESTORING FORCE, in that it works in the OPPOSITE direction of the displacement.





Sin or Cos function depends on the <u>starting position</u> of the SHM.

$$x = A\cos(2\pi ft)$$

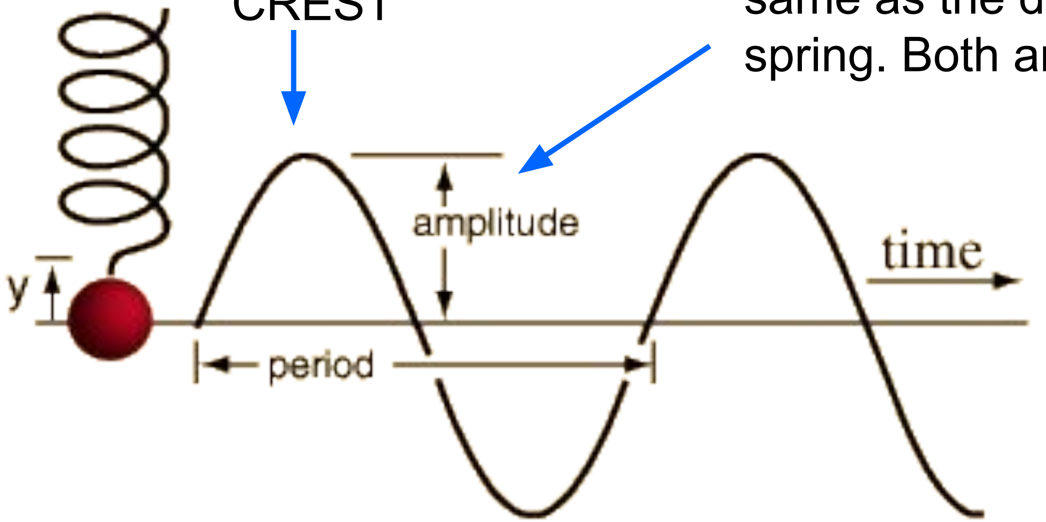
The choice of using a cosine in this equation is arbitrary. Other valid formulations are: $x = A\sin(\omega t + \varphi)$

The maximum displacement from equilibrium is the amplitude '*A*' of the motion. The shape of this graph is characteristic of simple harmonic motion and is called "*sinusoidal*," because it has the shape of a trigonometric sine or cosine function.

Springs are like Waves and Circles

The amplitude, A, of a wave is the same as the displacement ,x, of a spring. Both are in meters.

CREST



Equilibrium Line

Trough

T_s=sec/cycle. Let's assume that the wave crosses the equilibrium line in one second intervals. T =3.5 seconds/1.75 cycles. T = 2 sec.

Period, T, is the time for ONE COMPLETE oscillation (One crest and trough). Oscillations could also be called vibrations and cycles. In the wave above we have 1.75 cycles or waves or vibrations or oscillations.

Frequency

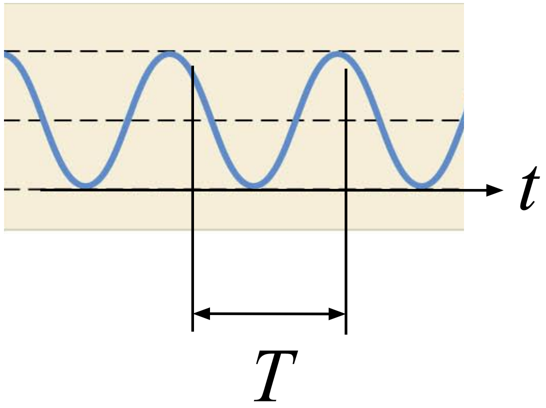
The FREQUENCY of a wave is the inverse of the PERIOD. That means that the frequency is the number of cycles per sec. The commonly used unit is HERTZ (Hz).

$$Period = T = \frac{\text{seconds}}{\text{cycles}} = \frac{3.5s}{1.75cyc} = 2s$$

$$Frequency = f = \frac{\text{cycles}}{\text{seconds}} = \frac{1.75cyc}{3.5\,\text{sec}} = 0.5\, {}^{c}\!/_{s} = 0.5Hz$$

$$T = \frac{1}{f} \quad f = \frac{1}{T}$$

The period and frequency of a wave



- the **period** T of a wave is the amount of time it takes to go through 1 cycle
- the **frequency** f is the number of cycles per second
- the unit of a cycle-per-second is called a **hertz** (Hz), after Heinrich Hertz (1847-1894), who discovered radio waves.
- frequency and period are related as follows:

$$f = \frac{1}{T}$$

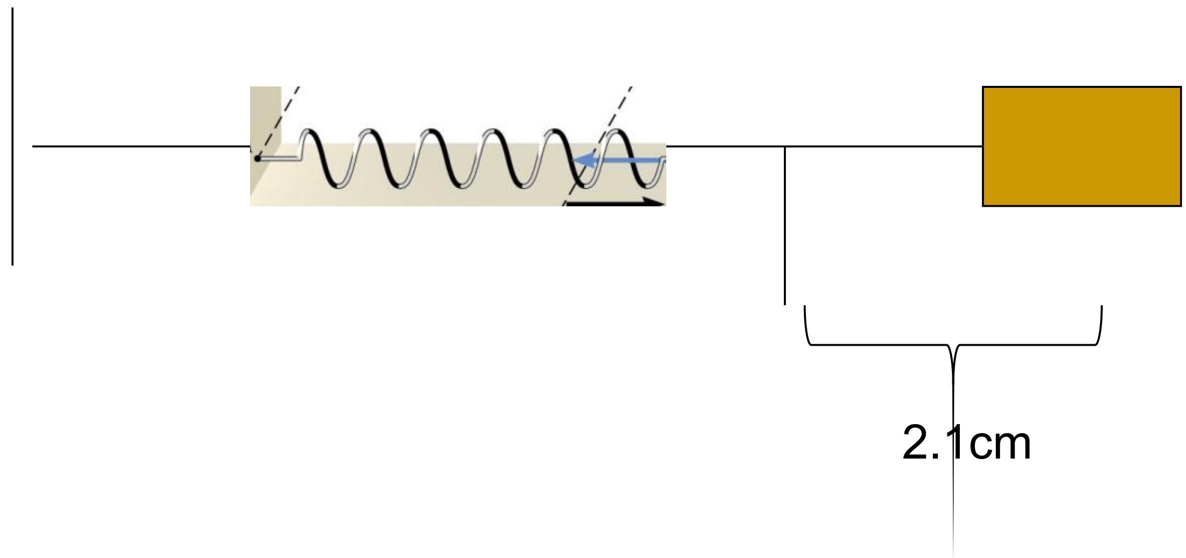
- Since a cycle is 2π radians, the relationship between frequency and angular frequency is:

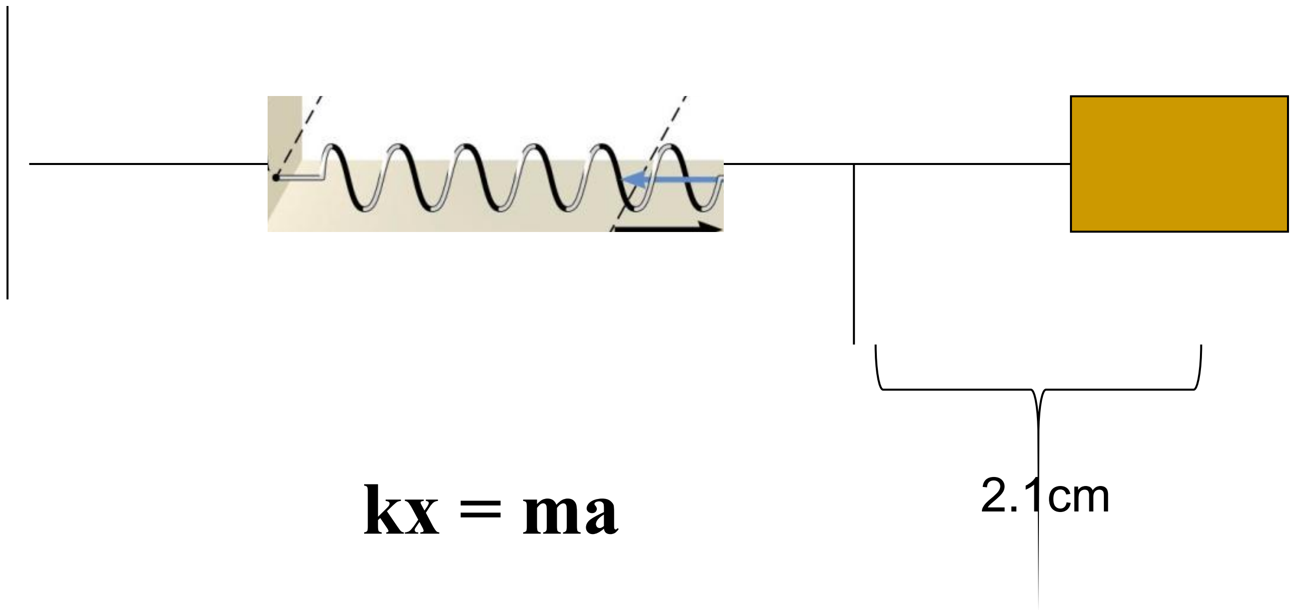
$$\omega = 2\pi f$$

HEINRICH RUDOLF HERTZ
1847 - 1894

Example: 1

A 0.42-kg block is attached to the end of a horizontal ideal spring and rests on a frictionless surface. The block is pulled so that the spring stretches by 2.1 cm relative to its unstrained length. When the block is released, it moves with an acceleration of 9.0 m/s^2. What is the spring constant of the spring?



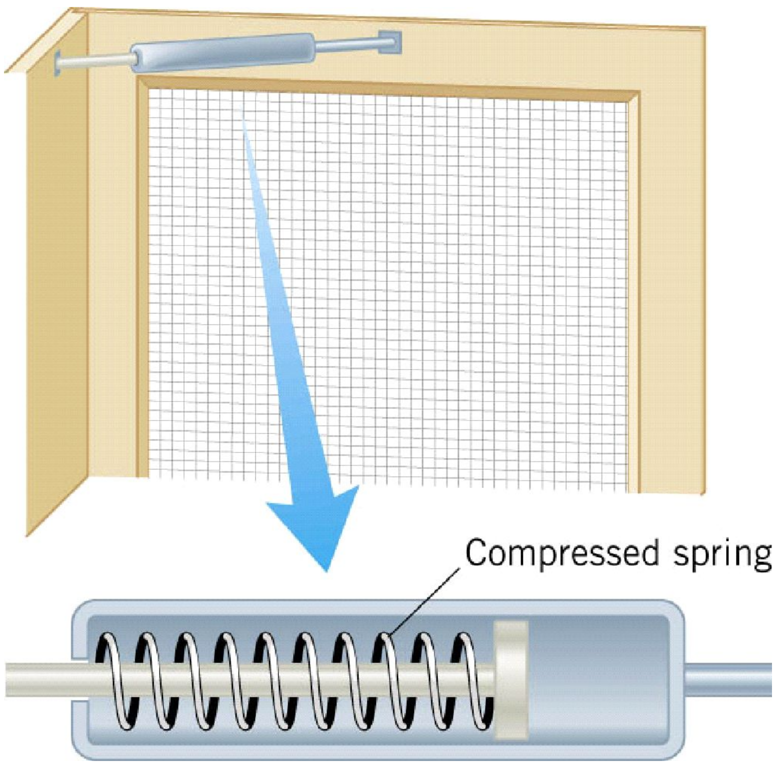


kx = ma

$$k \times \frac{2.1}{100} = 0.42 \times 9.0 m/s^2$$

$$k = \frac{0.42 \times 9.0}{2.1} \times 100 = 180 N/m$$

Energy and Simple Harmonic Motion



A spring also has potential energy when the spring is stretched or compressed, which we refer to as *elastic potential energy*. Because of elastic potential energy, a stretched or compressed spring can do work on an object that is attached to the spring.

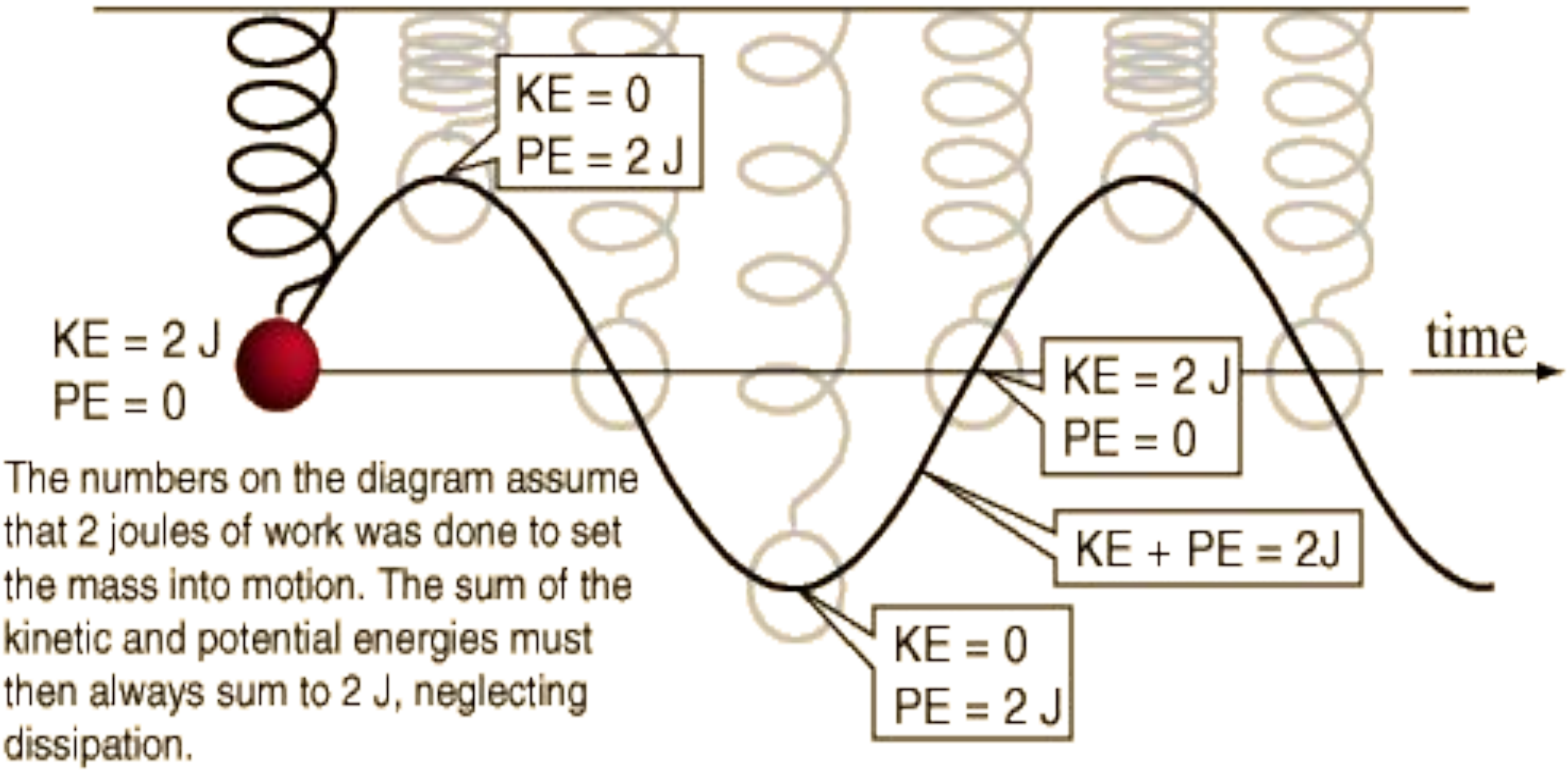
DEFINITION OF ELASTIC POTENTIAL ENERGY

The elastic potential energy $PE_{elastic}$ is the energy that a spring has by virtue of being stretched or compressed. For an ideal spring that has a spring constant *k* and is stretched or compressed by an amount *x* relative to its unstrained length, the elastic potential energy is

$$PE_{elastic} = \frac{1}{2} k x^2$$

***SI Unit of Elastic Potential Energy:* joule (J)**

Conservation of Energy in Springs



Example 2:

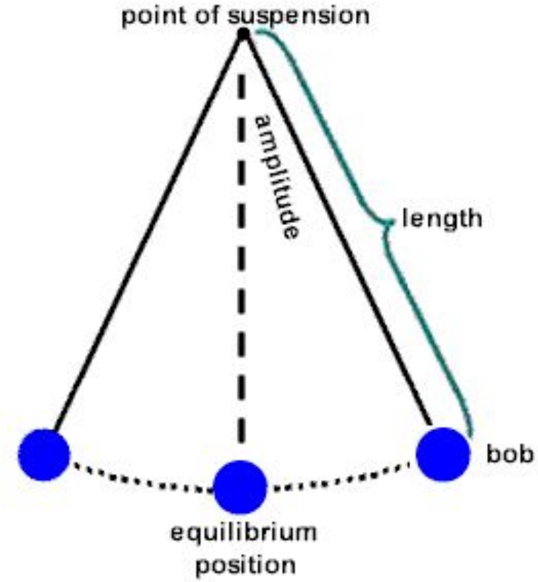
A 200 g mass is attached to a spring and executes simple harmonic motion with a period of 0.25 s If the total energy of the system is 2.0 J, find the (a) force constant of the spring (b) the amplitude of the motion

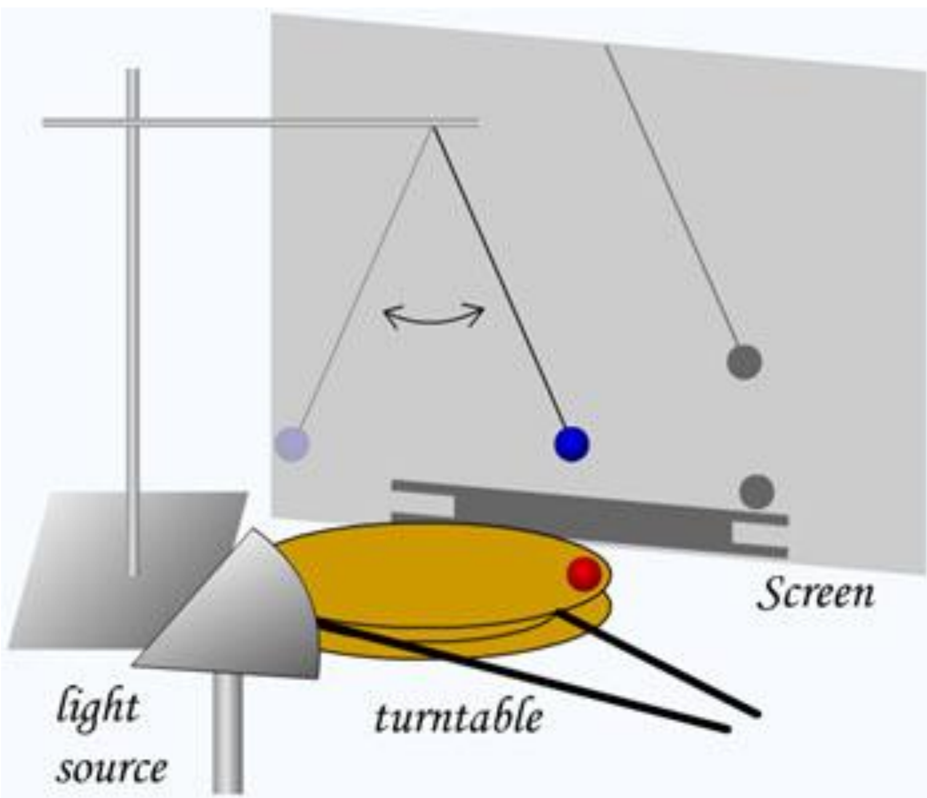
$$T_s = 2\pi\sqrt{\frac{m}{k}} \rightarrow 0.25 = 2\pi\sqrt{\frac{0.200}{k}} \qquad k = \quad \textbf{126.3 N/m}$$

$$U_s = \frac{1}{2}kx^2 \rightarrow 2 = \frac{1}{2}kA^2 \qquad A = \quad \textbf{0.18 m}$$

Pendulums

Pendulums, like springs, oscillate back and forth exhibiting simple harmonic behavior.





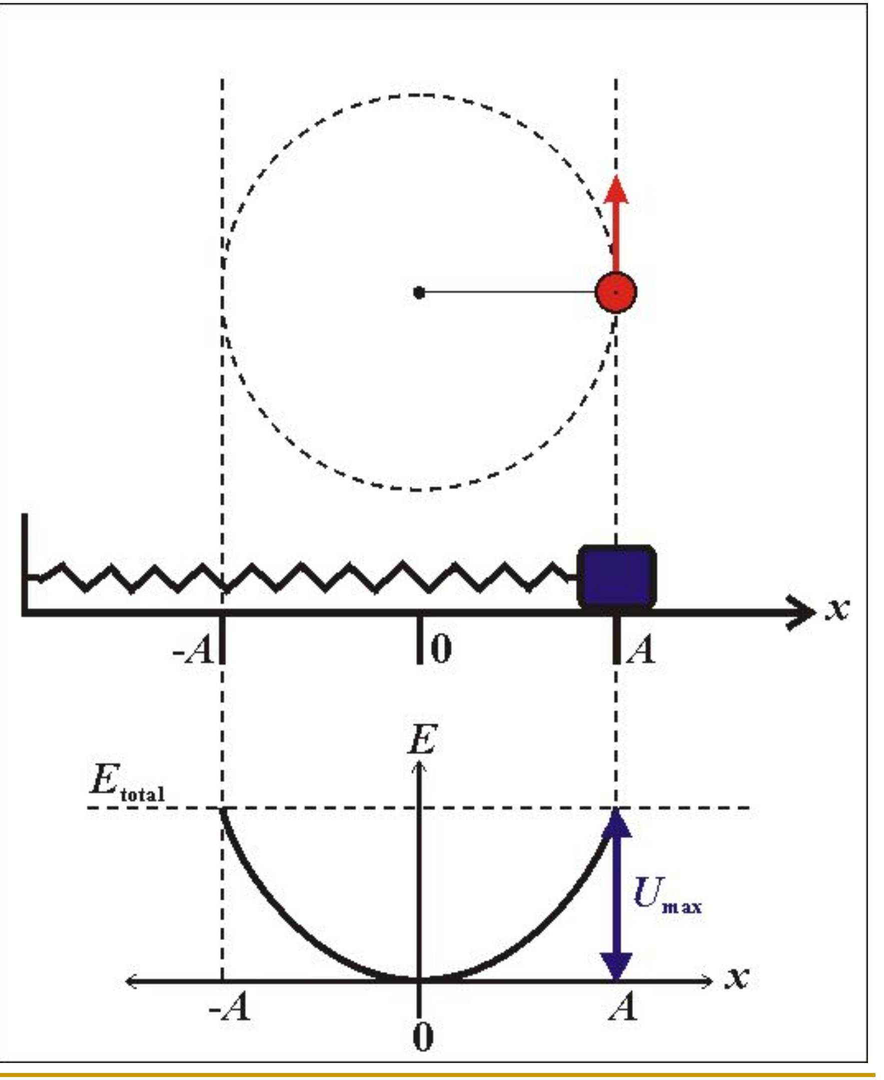
SHM is the projection of circular motion onto a screen. The shadow projector would show a pendulum moving in synchronization with a circle. Here, the angular amplitude is equal to the radius of a circle.

SHM and Uniform Circular Motion

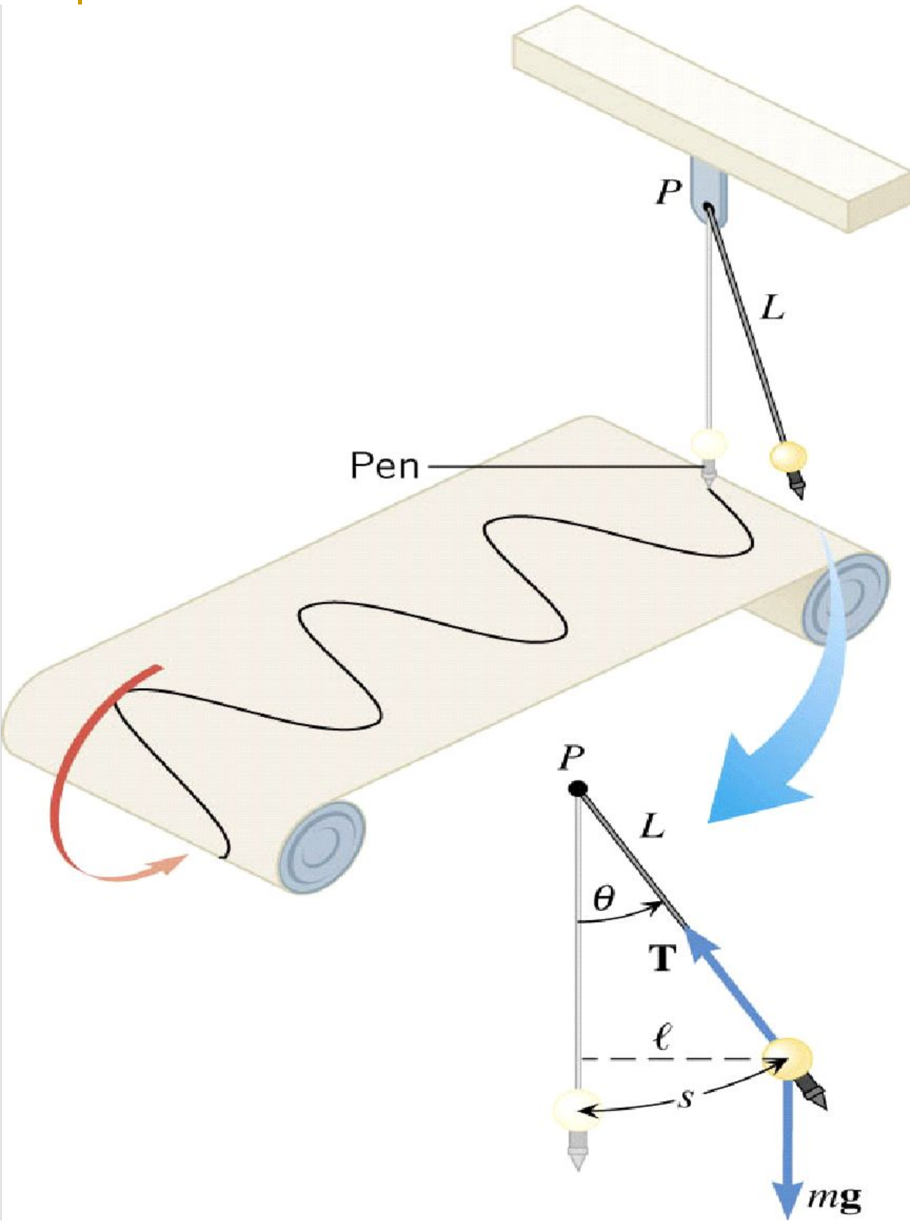
Springs and Waves behave very similar to objects that move in circles.

The radius of the circle is symbolic of the displacement, x, of a spring or the amplitude, A, of a wave.

$$x_{\max} = A_{wave} = r_{circle}$$



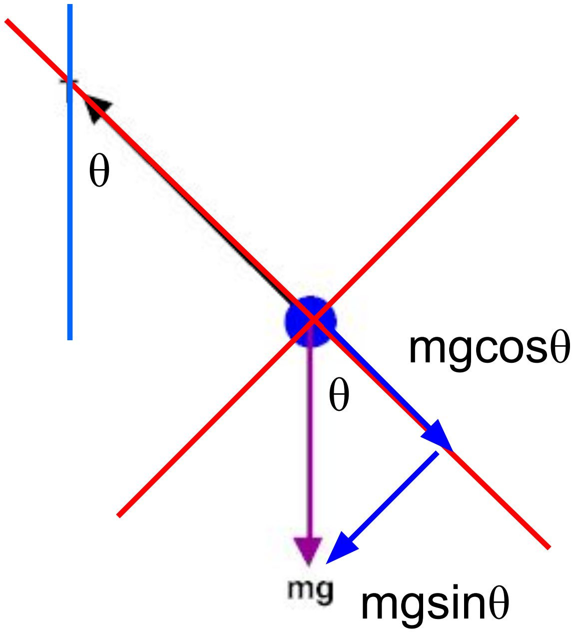
The Pendulum



A *simple pendulum* consists of a particle of mass *m*, attached to a frictionless pivot *P* by a cable of length *L* and negligible mass.

$$\tau \approx \underbrace{-mgL}_{k}\theta$$

Pendulums



Consider the free body diagram (FBD) for a pendulum. Here we have the weight and tension. Even though the weight isn't at an angle let's draw an axis along the tension.

$$mg\sin\theta = \text{Restoring Force}$$

$$mg\sin\theta = kx$$

Pendulums

$$mg\sin\theta = \text{Restoring Force}$$

$$mg\sin\theta = kx$$

$$\theta = \frac{s}{R} = \frac{s}{L}$$

$$s = \theta L = Amplitude$$

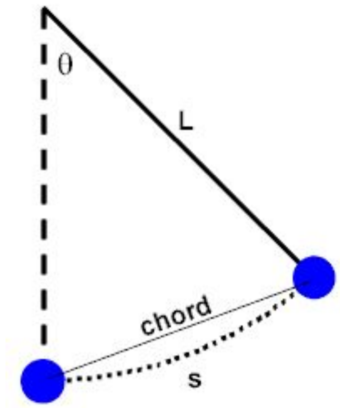
$$mg\sin\theta = k\theta L$$

$$\sin\theta \cong \theta, if\ \theta = small$$

$$mg = kl$$

$$\frac{m}{k} = \frac{l}{g}$$

$$T_{spring} = 2\pi\sqrt{\frac{m}{k}}$$



What is x? It is the amplitude! In the picture to the left, it represents the chord from where it was released to the bottom of the swing (equilibrium position).

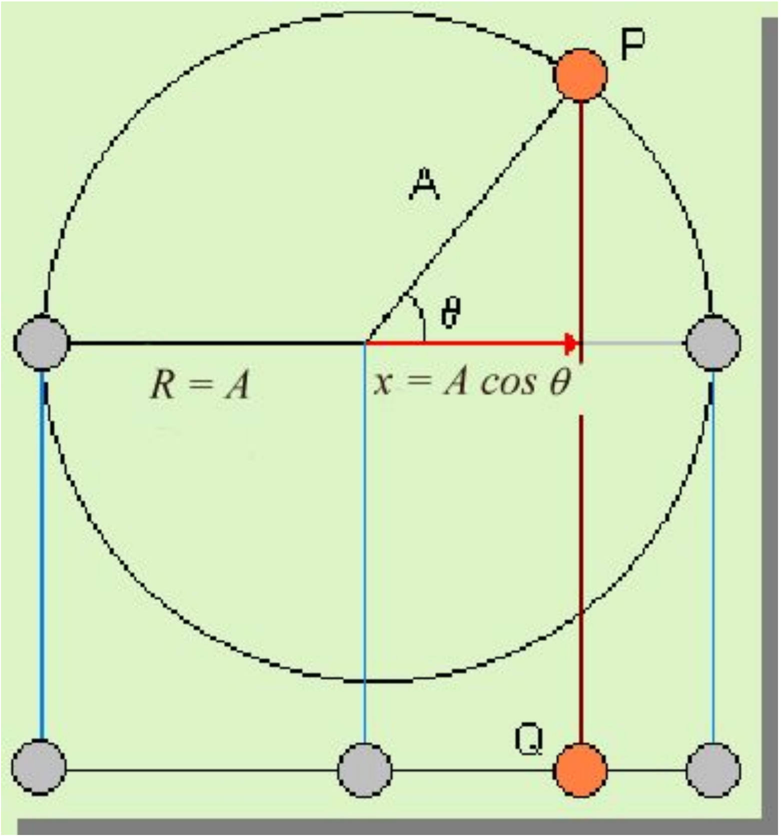
$$T_{pendulum} = 2\pi\sqrt{\frac{l}{g}}$$

The Reference Circle

The <u>reference circle</u> compares the circular motion of an object with its horizontal projection.

$$x = A\cos\theta \quad \theta = \omega t$$

$$x = A\cos(2\pi ft)$$



x = Horizontal displacement.

A = Amplitude (x_{max}).

θ = Reference angle.

$$\omega = 2\pi f$$

Velocity in SHM

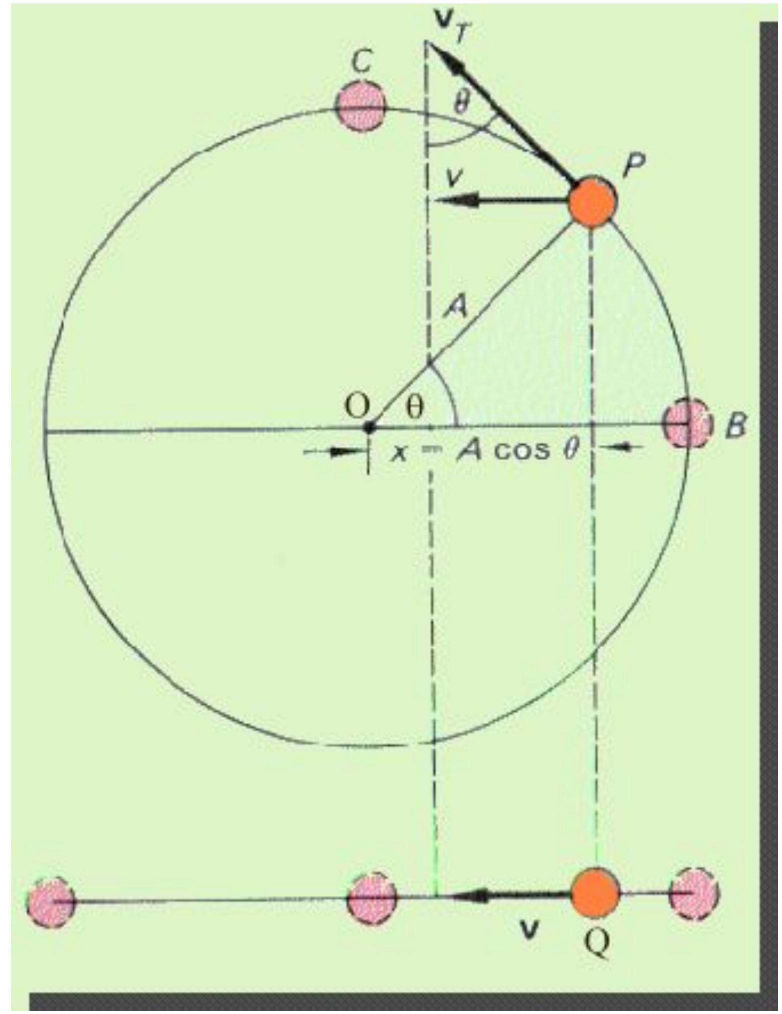
The <u>velocity</u> (v) of an oscillating body at any instant is the horizontal component of its tangential velocity (v_T).

$v_T = \omega R = \omega A; \quad \omega = 2\pi f$

$v = -v_T \sin \theta ; \quad \theta = \omega t$

$v = -\omega A \sin \omega t$

$v = -2\pi f A \sin 2\pi f t$



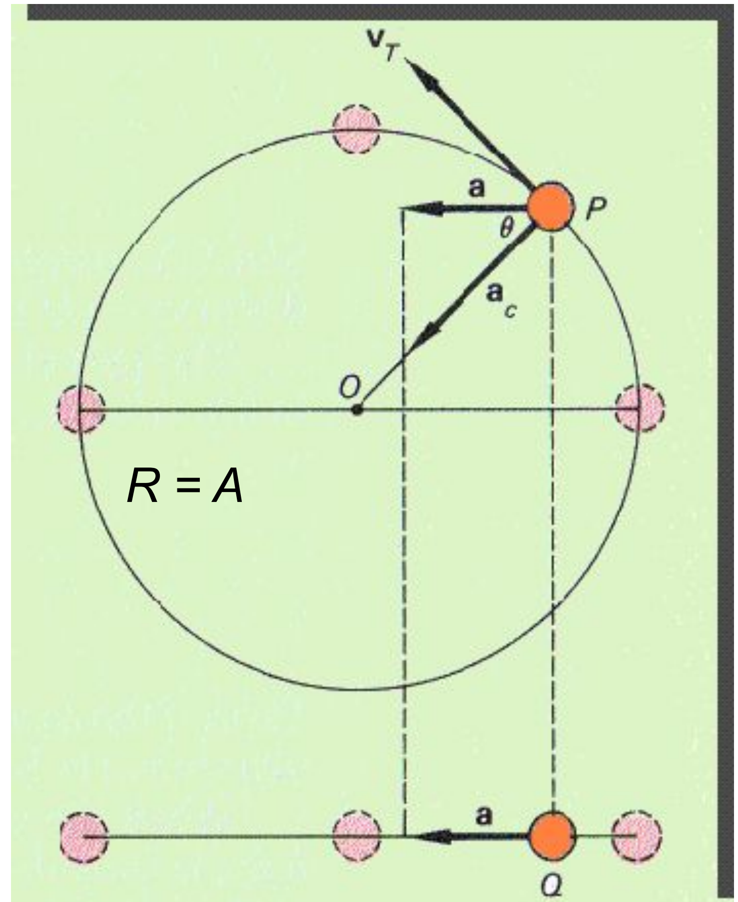
Acceleration Reference Circle

The <u>acceleration (*a*)</u> of an oscillating body at any instant is the horizontal component of its centripetal acceleration (a_c).

$$a = -a_c \cos\theta = -a_c \cos(\omega t)$$

$$a = -\omega^2 A \cos(\omega t)$$

$$a = -4\pi^2 f^2 x$$



The Period and Frequency as a Function of *a* and *x*.

For any body undergoing simple harmonic motion:

Since $a = -4\pi^2 f^2 x$ ***and*** $T = 1/f$

$$f = \frac{1}{2\pi}\sqrt{\frac{-a}{x}}$$

$$T = 2\pi\sqrt{\frac{-x}{a}}$$

The frequency and the period can be found if the displacement and acceleration are known. Note that the signs of *a* and *x* will always be opposite.

Period and Frequency as a Function of Mass and Spring Constant.

For a vibrating body with an elastic restoring force:

Recall that $F = ma = -kx$:

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

$$T = 2\pi\sqrt{\frac{m}{k}}$$

The frequency f and the period T can be found if the spring constant k and mass m of the vibrating body are known. Use consistent SI units.

Example 3:

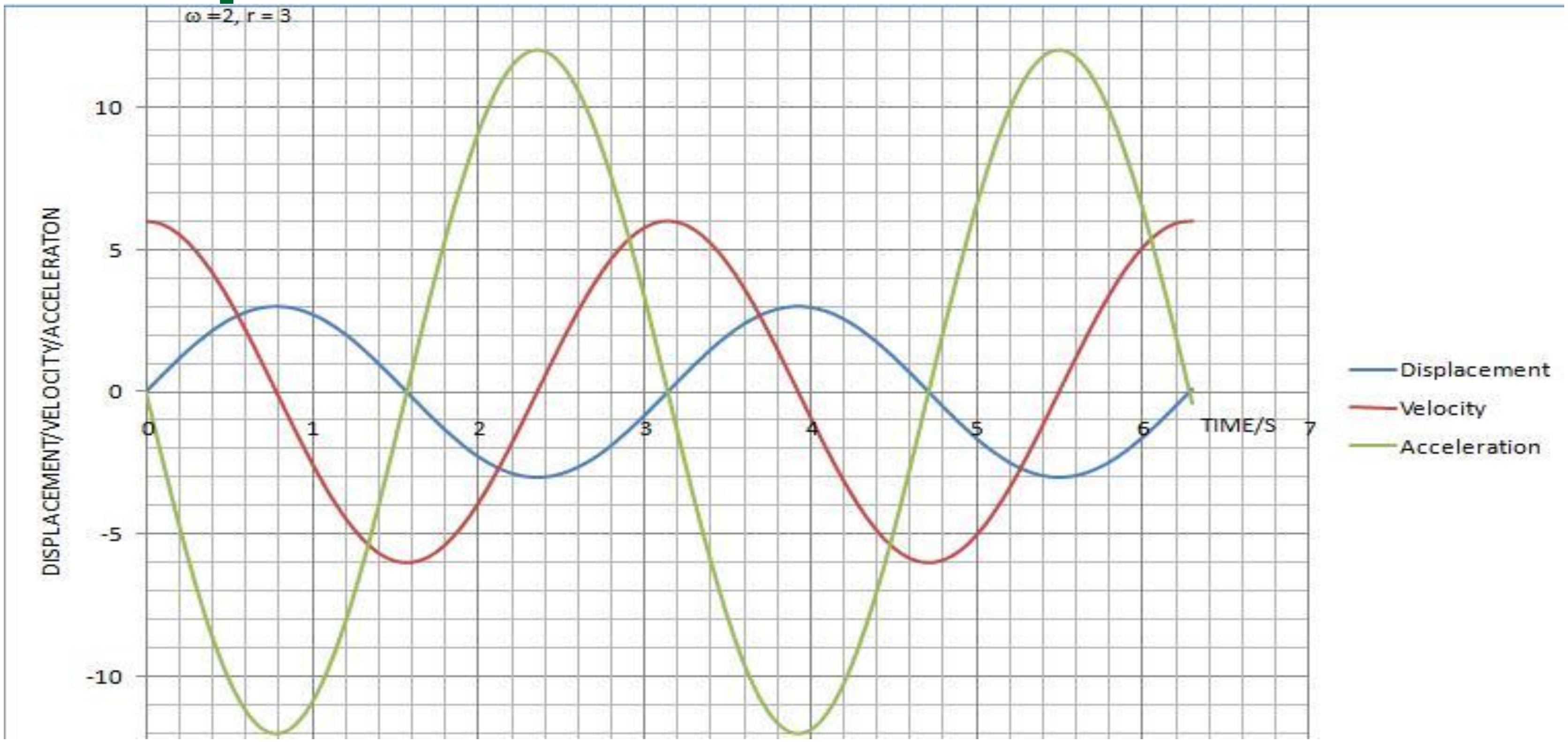
A visitor to a lighthouse wishes to determine the height of the tower. She ties a spool of thread to a small rock to make a simple pendulum, which she hangs down the center of a spiral staircase of the tower. The period of oscillation is 9.40 s. What is the height of the tower?

$$T_P = 2\pi\sqrt{\frac{l}{g}} \rightarrow l = height$$

$$T_P^2 = \frac{4\pi^2 l}{g} \rightarrow l = \frac{T_P^2 g}{4\pi^2} = \frac{9.4^2(9.8)}{4(3.141592)^2} =$$

L = Height = 21.93 m

Graphical Analysis of Simple Harmonic Motion



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Objectives:

- State the formula required for SHM.
- Recognize equations and graphs which represent the variation of displacement, velocity and acceleration with time.
- Investigate simulations of mass-spring systems and the simple pendulum.

Oscillation of the spring and the motion of the les of simple harmonic motion.

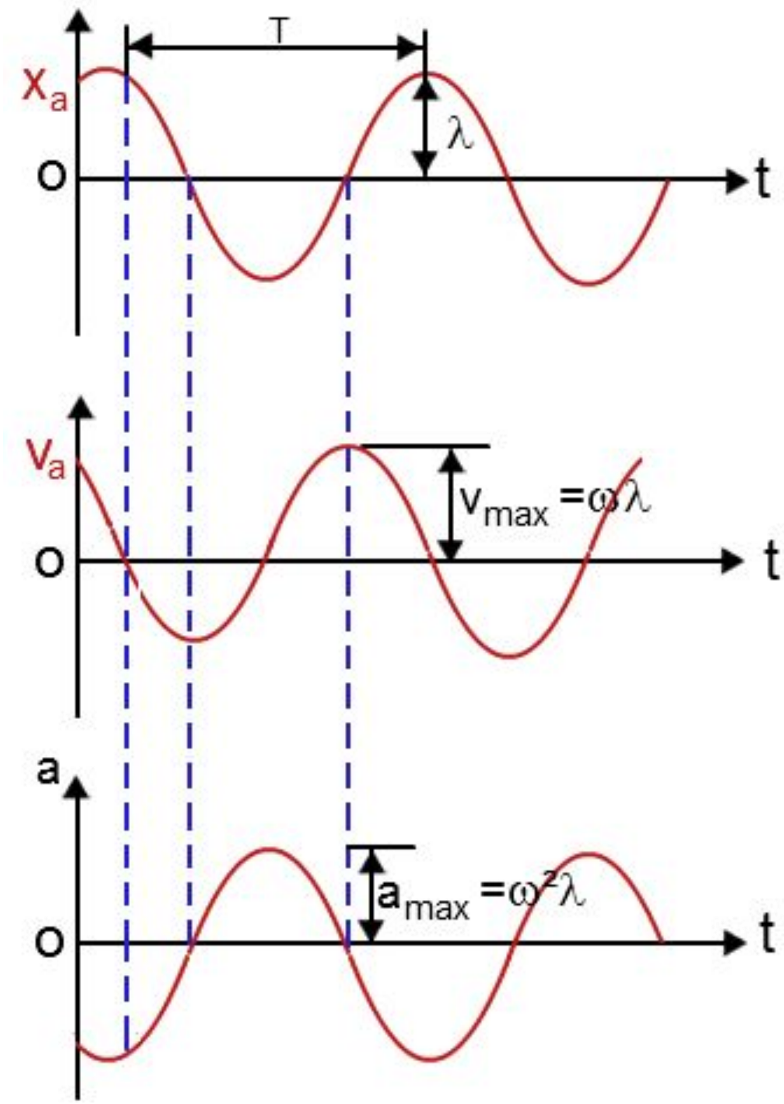

fig a is showing the displacement time graph

fig b is showing the velocity time graph

fig c is showing the acceleration time graph of the Simple Harmonic Motion.

Simple Harmonic Motion References

- http://www.physics.uoguelph.ca/tutorials/shm/Q.shm.html
- http://www.youtube.com/watch?v=bPtlRf6dg8c

The Fizzics Organization: A level tutorials and Your tube play list:

- https://www.youtube.com/playlist?list=PLLKB_7Zd6lePXgOD1QBgl3d6D14x6mc_p

Simple Harmonic Motion @ www.ThePhysicsCafe.com

- http://www.youtube.com/watch?v=SZ541Luq4nE

Many slides have been modified from Paul E.Tippens, Professor of Physics, Southern Polytechnic State University :

- https://www.google.me/url?sa=t&rct=j&q=&esrc=s&source=web&cd=1&cad=rja&uact=8&ved=0CBsQFjAA&url=http%3A%2F%2Fhighered.mheducation.com%2Fsites%2Fdl%2Ffree%2F007301267x%2F294279%2FChapter14.ppt&ei=C6x_VJ6FIYL_ygPM-YKQCA&usg=AFQjCNGa4CRKUgdcYzFFfad6QWOfq_NAsQ&bvm=bv.80642063,d.bGQ